AS-4046 Solution B.Tech. III Sem Exam - 2013 (odd Sem.)

Network Theory

Section - A

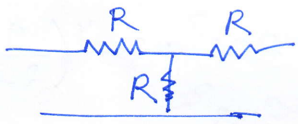
(i) damping ratio δ $(0<\delta<1)$ (c) complex conjugate

(ii) e^{at} $\longrightarrow$ (b) $1/(s-a)$

(iii) L_{AB} $\longrightarrow$ (a) $L_1+L_2+L_3+2M_{12}-2M_{23}-2M_{31}$

(iv) V, Z $\longrightarrow$ (c) 100, 30

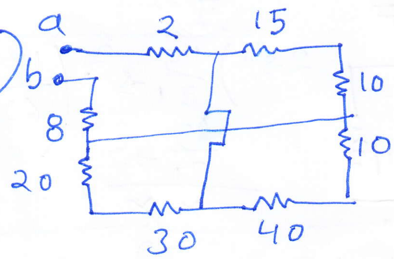
(v) $Y=\begin{bmatrix} 0 & -1/2 \\ 1/2 & 0 \end{bmatrix} \rightarrow$ (a) Non reciprocal and passive

(vi) 

$h_{21} \rightarrow$ (a) $-1/2$

(vii) $1/s$: $\longrightarrow$ (d) can't be realized by an R-L-C N/w

(viii) transfer $f^n \rightarrow$ (a) The coefficient in polynomial $P(s)$ and $Q(s)$ must be real

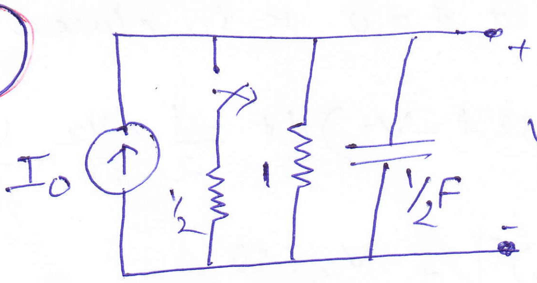
(ix) 

$R_{ab} \rightarrow$ (a) 22.5

(x) $P(s) = s^7+2s^6+2s^5+s^4+4s^3+8s^2+8s+4$

$\rightarrow$ Not Hurwitz

Unit- I

(2) 

K is opened at $t=0$ find $V_2(t)$

at $t=0^-$ $\Rightarrow V_2 = \frac{1}{2} \| 1 \times I_0 = \frac{I_0}{3}$

for $t \geq 0$

$$I_0 = \frac{V_2}{1} + \frac{1}{2}\frac{dV_2}{dt}$$

$$\Rightarrow \frac{dV_2}{dt} + 2V_2 = 2I_0$$

$$\Rightarrow V_2 = I_0 + Ce^{-2t}$$

for $t=0^+$ $\Rightarrow \frac{I_0}{3} = I_0 + C \cdot 1 \Rightarrow C = -\frac{2}{3}I_0$

$$\Rightarrow \boxed{V_2 = I_0\left(1 - \frac{2}{3}e^{-2t}\right)}$$

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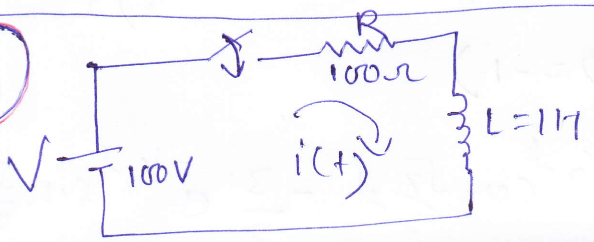
(3) 

K is closed at $t=0$

$$V = Ri + L\frac{di}{dt}$$

$$i = \frac{V}{R} - \frac{V}{R}e^{-R/L\, t}$$

$$\Rightarrow \boxed{i = 10 - 10e^{-10t}}$$

$\boxed{i(0^+) = 0A}$, $\boxed{\frac{di}{dt}(0^+) = 0 + 100e^{-10t} = 100 A/s}$, $\boxed{\frac{d^2i}{dt^2}(0^+) = -1000 A/s^2}$

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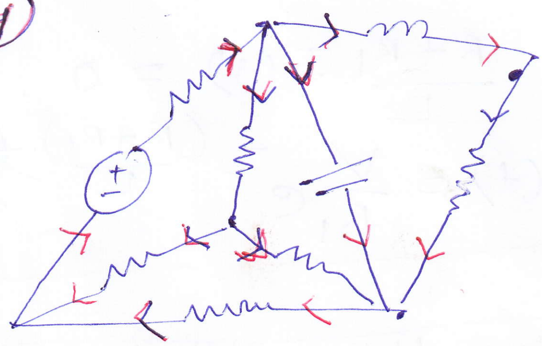
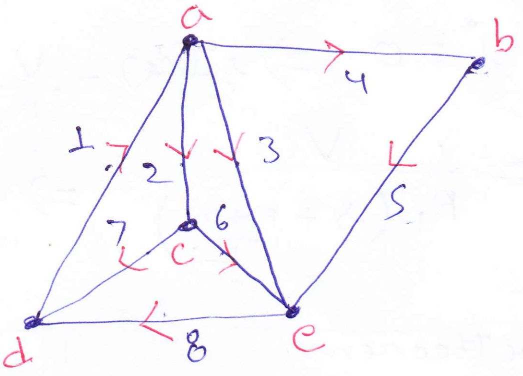
(4) 

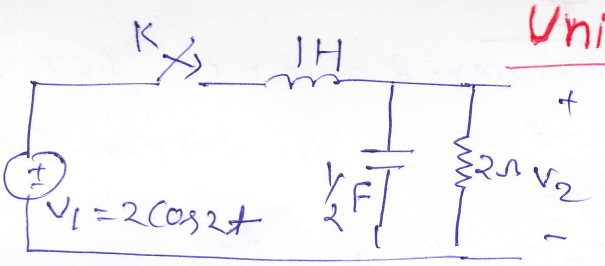


Incident Matrix A =

$$\begin{array}{c|cccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline a & -1 & +1 & +1 & +1 & 0 & 0 & 0 & 0 \\ b & 0 & 0 & 0 & -1 & +1 & 0 & 0 & 0 \\ c & 0 & -1 & 0 & 0 & 0 & +1 & +1 & 0 \\ d & +1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \\ e & 0 & 0 & -1 & 0 & -1 & -1 & 0 & +1 \end{array}$$

Unit - II

(5)



at $t=0$ K is closed

$$\frac{1}{1}\int(2\cos 2t - V_2)\,dt = \frac{1}{2}\frac{dV_2}{dt} + \frac{V_2}{2}$$

$$\Rightarrow 2\frac{s}{s^2+4}\times\frac{1}{s} - \frac{V_2(s)}{s} = \frac{1}{2}\left[sV_2(s)\right] + \frac{V_2(s)}{2}$$

$$\Rightarrow V_2(s)\left[\frac{s^2+s+2}{2s}\right] = \frac{2}{s^2+4} \Rightarrow V_2(s) = \frac{4s}{(s^2+4)(s^2+s+2)}$$

$$V_2(s) = \frac{As+B}{(s^2+4)} + \frac{Cs+D}{s^2+s+2} = \frac{-s}{s^2+4} + \frac{2}{s^2+4} + \frac{s-1}{(s+\frac{1}{2})^2+(\frac{\sqrt{7}}{2})^2}$$

$(A=-1,\ B=2,\ C=1,\ D=-1)$

$$V_2(t) = -\cos 2t + \sin 2t + e^{-\frac{1}{2}t}\cos\frac{\sqrt{7}}{2}t - \frac{3}{2}e^{-\frac{1}{2}t}\sin\frac{\sqrt{7}}{2}t$$

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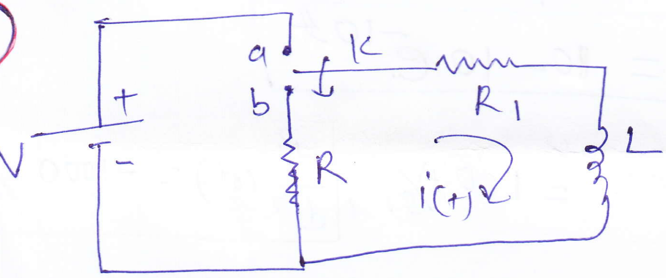
(6)



S.S. at a, K: a → b at $t=0$

find $i(t)$ for $t \geq 0$, with LT.

$i(0^-) = V/R_1$

$$R_1 i + L\frac{di}{dt} + Ri = 0$$

$$\Rightarrow \frac{di}{dt} + \frac{R+R_1}{L}i = 0 \Rightarrow sI(s) - \frac{V}{R_1} + \frac{R+R_1}{L}I(s) = 0$$

$$\Rightarrow I(s) = \frac{V}{R_1\left(s+\frac{R+R_1}{L}\right)} \Rightarrow \boxed{i(t) = \frac{V}{R_1}e^{-\left(\frac{R+R_1}{L}\right)t}}$$

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(7) Initial value Theorem

$$f(0^+) = \lim_{t\to 0} f(t) = \lim_{s\to\infty}(s\cdot F(s))$$

Final value Theorem

$$f(\infty) = \lim_{t\to\infty} f(t) = \lim_{s\to 0}(s\cdot F(s)) \rightarrow \text{Put} \lim_{s\to 0}$$

Using time-differentiation

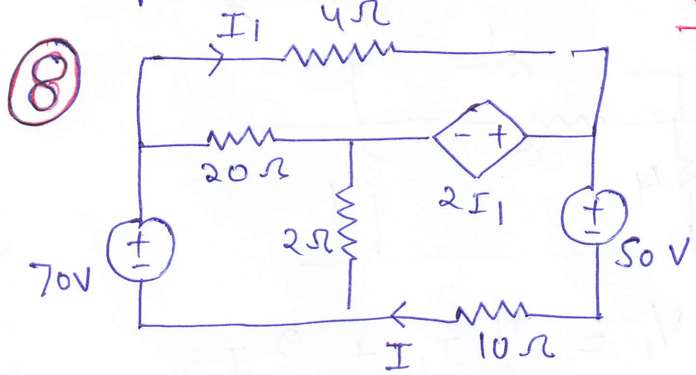
$$s\cdot F(s) - f(0^+) = \int_0^\infty \frac{df(t)}{dt}e^{-st}\,dt$$

→ Put $\lim_{s\to\infty}$

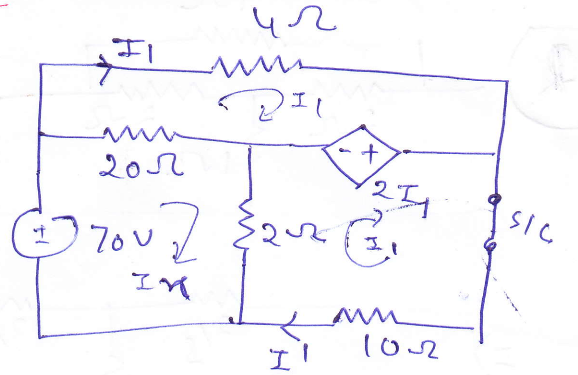
Unit - III

Superposition

(8)



Case-I



$$70 - 20(I_n - I_1) - 2(I_n - I') = 0 \quad \text{—(I)}$$

$$2I - 10I' - 2(I' - I_n) = 0 \quad \text{—(II)}$$

$$-4I_1 - 2I_1 - 20(I_1 - I_n) = 0 \quad \text{—(III)}$$

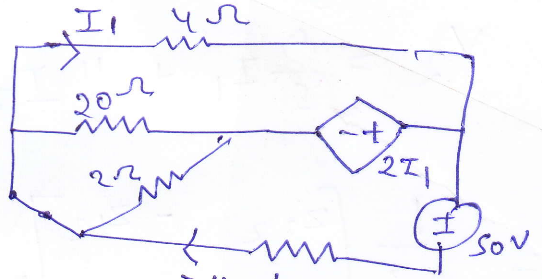
$$I' = 3.425\ A$$

hence $I = I' + I'' = -1.15A$

and hence $I = I' + I'' = 8\ A$ (with ∓50V)

Case-II

$20 \| 2 = \frac{20}{11}$

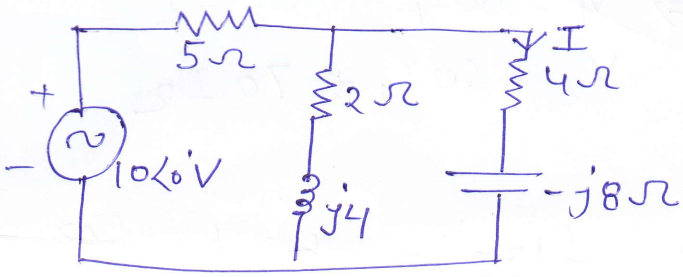


$$\Rightarrow 50 - 2I_1 - \frac{20}{11}(I_1 - I'') + 10I'' = 0 \quad \text{—(I)}$$

$$\Rightarrow -4I_1 - 2I_1 - \frac{20}{11}(I_1 - I'') = 0 \quad \text{—(II)}$$

$$\Rightarrow I'' = -4.575$$

(9)



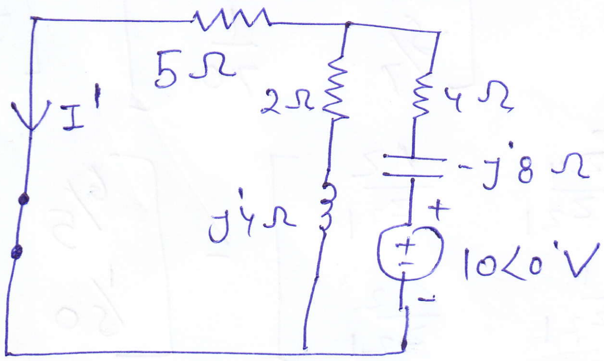
Case-I

$$I = \frac{10\angle 0}{5 + (2+4j) \| (4-8j)} \times \frac{(2+4j)}{(2+4j)+(4-j8)}$$

$$= \frac{10\angle 0}{10.1\angle 17.75} \times \frac{4.47\angle 63.4}{7.21\angle -33.65}$$

$$= 0.614\angle 79.3°\ A$$

Case-II

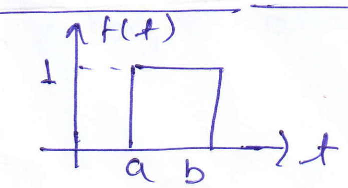


$$I' = \frac{10\angle 0}{4 - j8 + 5 \| (2+4j)} \times \frac{2+4j}{(2+4j)+5}$$

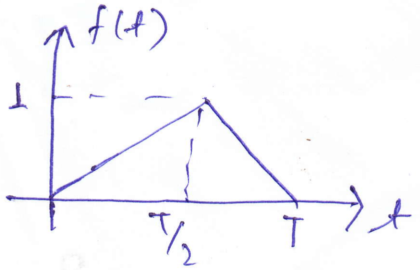
$$= 0.614\angle 79.3°\ A$$

hence reciprocal.

(10)



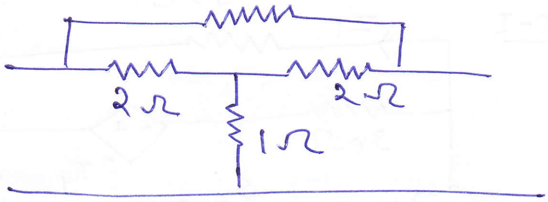
$$f(t) = u(t-a) - u(t-b)$$



$$f(t) = \frac{2}{T} r(t) - \frac{4}{T} r(t - T/2) + \frac{2}{T} r(t - T)$$

Unit-IV

12



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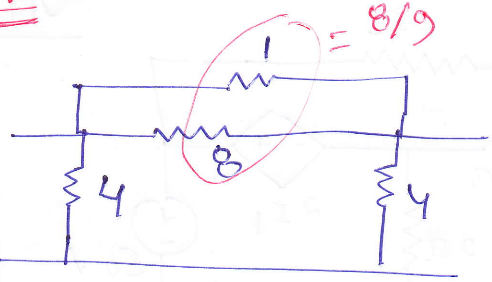
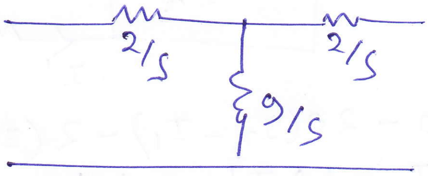


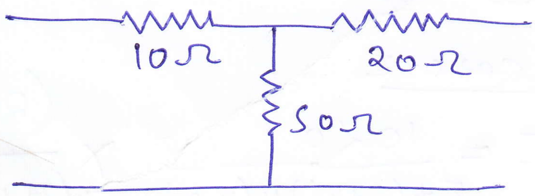
$\Rightarrow$



$$\Rightarrow \quad V_1 = \frac{11}{5} I_1 + \frac{9}{5} I_2$$
$$V_2 = \frac{9}{5} I_1 + \frac{11}{5} I_2$$

$$Z = \begin{bmatrix} 11/5 & 9/5 \\ 9/5 & 11/5 \end{bmatrix}, \quad Y = \frac{1}{\Delta Z}\begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix} = \begin{bmatrix} \frac{11}{8} & -\frac{9}{8} \\ -\frac{9}{8} & \frac{11}{8} \end{bmatrix}$$

12



$$\Rightarrow V_1 = 60 I_1 + 50 I_2$$
$$V_2 = 50 I_1 + 70 I_2$$

$$\Rightarrow V_1 = \frac{170}{7} I_1 + \frac{5}{7} V_2$$
$$I_2 = -\frac{5}{7} I_1 + \frac{1}{70} V_2$$

$$\Rightarrow h = \begin{bmatrix} \frac{170}{7} & \frac{5}{7} \\ -\frac{5}{7} & \frac{1}{70} \end{bmatrix} = \begin{bmatrix} \frac{\Delta z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix}$$

$$T = \begin{bmatrix} \frac{\Delta h}{h_{21}} & -\frac{h_{11}}{h_{21}} \\ -\frac{h_{22}}{h_{21}} & -\frac{1}{h_{21}} \end{bmatrix} = \begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix} = \begin{bmatrix} 6/5 & 34 \\ 1/50 & 7/5 \end{bmatrix}$$

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13 Stability of N/w based on location of zeros & poles.

1 System Poles and Zeros

The transfer function provides a basis for determining important system response characteristics without solving the complete differential equation. As defined, the transfer function is a rational function in the complex variable $s = \sigma + j\omega$, that is

$$H(s) = \frac{b_m s^m + b_{m-1}s^{m-1} + \ldots + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0} \tag{1}$$

It is often convenient to factor the polynomials in the numerator and denominator, and to write the transfer function in terms of those factors:

$$H(s) = \frac{N(s)}{D(s)} = K\frac{(s-z_1)(s-z_2)\ldots(s-z_{m-1})(s-z_m)}{(s-p_1)(s-p_2)\ldots(s-p_{n-1})(s-p_n)}, \tag{2}$$

where the numerator and denominator polynomials, $N(s)$ and $D(s)$, have real coefficients defined by the system's differential equation and $K = b_m/a_n$. As written in Eq. (2) the z_i's are the roots of the equation

$$N(s) = 0, \tag{3}$$

and are defined to be the system *zeros*, and the p_i's are the roots of the equation

$$D(s) = 0, \tag{4}$$

and are defined to be the system *poles*. In Eq. (2) the factors in the numerator and denominator are written so that when $s = z_i$ the numerator $N(s) = 0$ and the transfer function vanishes, that is

$$\lim_{s \to z_i} H(s) = 0.$$

and similarly when $s = p_i$ the denominator polynomial $D(s) = 0$ and the value of the transfer function becomes unbounded,

$$\lim_{s \to p_i} H(s) = \infty.$$

All of the coefficients of polynomials $N(s)$ and $D(s)$ are real, therefore the poles and zeros must be either purely real, or appear in complex conjugate pairs. In general for the poles, either $p_i = \sigma_i$, or else $p_i, p_{i+1} = \sigma_i \pm j\omega_i$. The existence of a single complex pole without a corresponding conjugate pole would generate complex coefficients in the polynomial $D(s)$. Similarly, the system zeros are either real or appear in complex conjugate pairs.

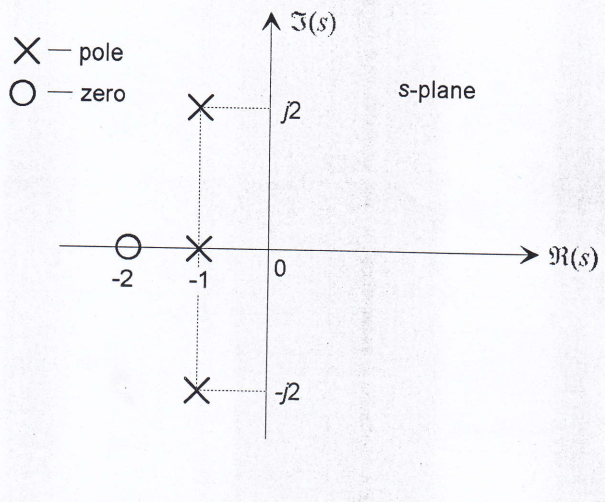


Figure 1: The pole-zero plot for a typical third-order system with one real pole and a complex conjugate pole pair, and a single real zero.

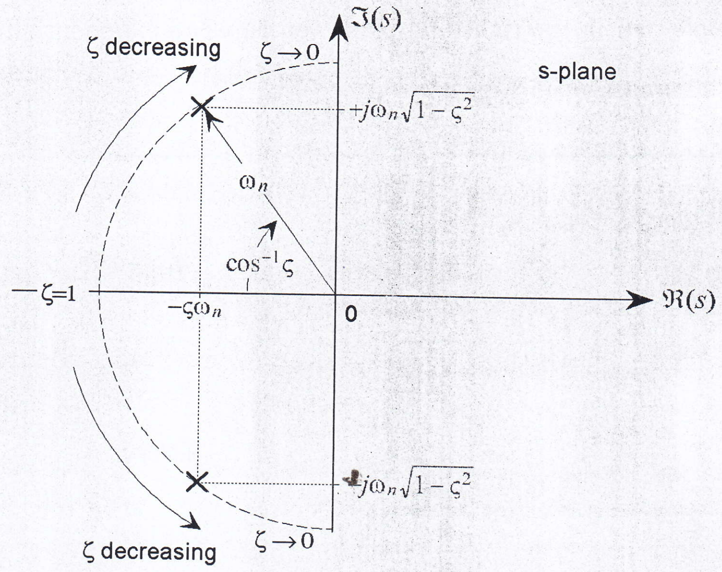


Figure 4: Definition of the parameters ω_n and ζ for an underdamped, second-order system from the complex conjugate pole locations.

The pole locations of the classical second-order homogeneous system

$$\frac{d^2y}{dt^2} + 2\zeta\omega_n \frac{dy}{dt} + \omega_n^2 y = 0, \tag{13}$$

described in Section 9.3 are given by

$$p_1, p_2 = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}. \tag{14}$$

If $\zeta \geq 1$, corresponding to an overdamped system, the two poles are real and lie in the left-half plane. For an underdamped system, $0 \leq \zeta < 1$, the poles form a complex conjugate pair,

$$p_1, p_2 = -\zeta\omega_n \pm j\omega_n\sqrt{1 - \zeta^2} \tag{15}$$

and are located in the left-half plane, as shown in Fig. 4. From this figure it can be seen that the poles lie at a distance ω_n from the origin, and at an angle $\pm\cos^{-1}(\zeta)$ from the negative real axis. The poles for an underdamped second-order system therefore lie on a semi-circle with a radius defined by ω_n, at an angle defined by the value of the damping ratio ζ.

1.3 System Stability

The stability of a linear system may be determined directly from its transfer function. An nth order linear system is asymptotically stable only if all of the components in the homogeneous response from a finite set of initial conditions decay to zero as time increases, or

$$\lim_{t\to\infty} \sum_{i=1}^{n} C_i e^{p_i t} = 0. \tag{16}$$

where the p_i are the system poles. In a stable system all components of the homogeneous response must decay to zero as time increases. If any pole has a positive real part there is a component in the output that increases without bound, causing the system to be unstable.

4

In order for a linear system to be stable, all of its poles must have negative real parts, that is they must all lie within the left-half of the s-plane. An "unstable" pole, lying in the right half of the s-plane, generates a component in the system homogeneous response that increases without bound from any finite initial conditions. A system having one or more poles lying on the imaginary axis of the s-plane has non-decaying oscillatory components in its homogeneous response, and is defined to be *marginally* stable.

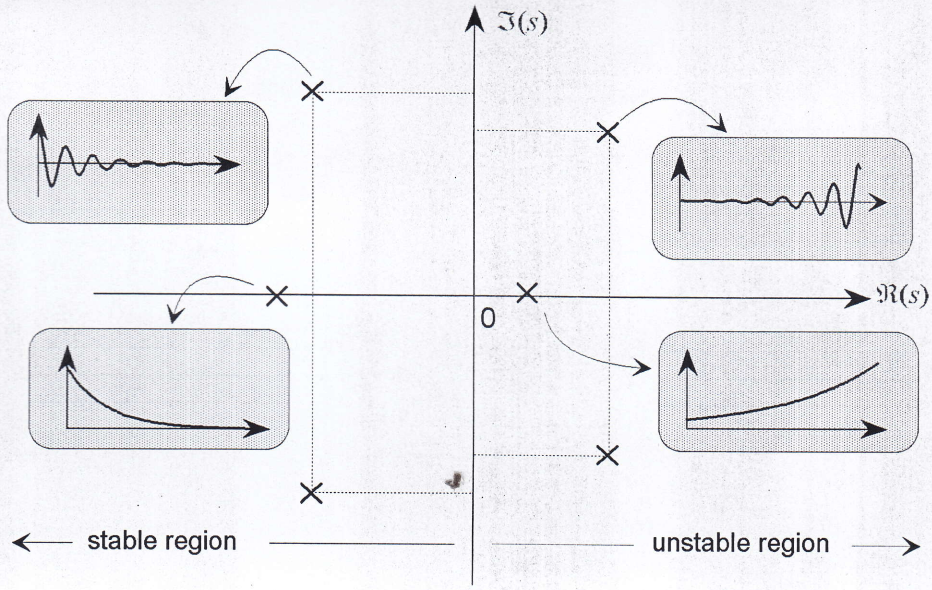


Figure 2: The specification of the form of components of the homogeneous response from the system pole locations on the pole-zero plot.

> The transfer function poles are the roots of the characteristic equation, and also the eigenvalues of the system **A** matrix.

The homogeneous response may therefore be written

$$y_h(t) = \sum_{i=1}^{n} C_i e^{p_i t}. \tag{11}$$

The location of the poles in the s-plane therefore define the n components in the homogeneous response as described below:

1. A real pole $p_i = -\sigma$ in the left-half of the s-plane defines an exponentially decaying component , $Ce^{-\sigma t}$, in the homogeneous response. The rate of the decay is determined by the pole location; poles far from the origin in the left-half plane correspond to components that decay rapidly, while poles near the origin correspond to slowly decaying components.
2. A pole at the origin $p_i = 0$ defines a component that is constant in amplitude and defined by the initial conditions.
3. A real pole in the right-half plane corresponds to an exponentially increasing component $Ce^{\sigma t}$ in the homogeneous response; thus defining the system to be unstable.
4. A complex conjugate pole pair $\sigma \pm j\omega$ in the left-half of the s-plane combine to generate a response component that is a decaying sinusoid of the form $Ae^{-\sigma t}\sin(\omega t + \phi)$ where A and ϕ are determined by the initial conditions. The rate of decay is specified by σ; the frequency of oscillation is determined by ω.
5. An imaginary pole pair, that is a pole pair lying on the imaginary axis, $\pm j\omega$ generates an oscillatory component with a constant amplitude determined by the initial conditions.

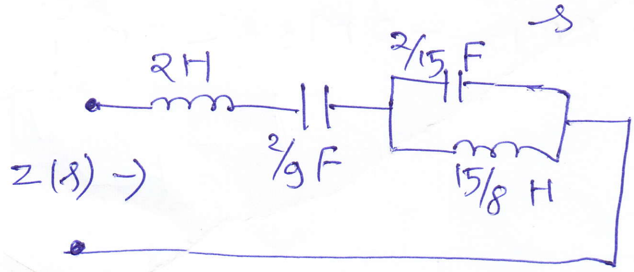
Sol^n:- (14) $P(s) = (s^2+2)(s^2+3)(s+2)(s+3)$

Since above equation have ~~All~~ roots (poles) with ~~+ve~~/zero real parts ie:- $(\pm\sqrt{2}j$ & $\pm\sqrt{3}j), (-2,-3)$

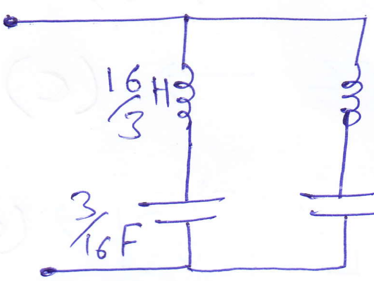
hence given eq^n is ~~not~~ Hurwitz.

Sol^n:- (15) $Z(s) = \dfrac{2(s^2+1)(s^2+9)}{s(s^2+4)} = \dfrac{2s^4+20s^2+18}{s^3+4s}$

F-I $Z(s) = 2s + \dfrac{\frac{9}{2}}{s} + \dfrac{\frac{15}{2}s}{s^2+4}$

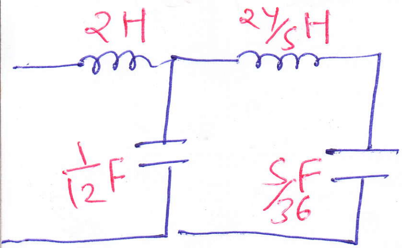


F-II $Y(s) = \dfrac{\frac{3}{16}s}{s^2+1} + \dfrac{\frac{5}{16}s}{s^2+9}$



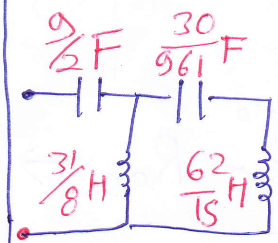
C-I

$s^3+4s \,)\, 2s^4+20s^2+18 \,(\, 2s \to Z_1$
$\quad 2s^4+8s^2$
$12s^2+18 \,)\, s^3+4s \,(\, \frac{1}{12}s \to Y_2$
$\quad s^3+\frac{3}{2}s$
$\frac{5}{2}s \,)\, 12s^2+18 \,(\, \frac{24}{5}s$
$\quad 12s^2$
$18 \,)\, \frac{5}{2}s \,(\, \frac{5}{36}s$
$\quad \frac{5}{2}s$
$\quad \times\times$



C-II

$4s+s^3 \,)\, 18+20s^2+2s^4 \,(\, \frac{9}{2s} \to Z_1$
$\quad 18+\frac{9}{2}s^2$
$\frac{31}{2}s^2+2s^4 \,)\, 4s+s^3 \,(\, \frac{8}{31s} \to Y_2$
$\quad 4s+\frac{16}{31}s^3$
$\frac{15}{31}s^3 \,)\, \frac{31}{2}s^2+2s^4 \,(\, \frac{961}{30s}$
$\quad \frac{31}{2}s^2$
$2s^4 \,)\, \frac{15}{31}s^3 \,(\, \frac{15}{62s}$
$\quad \frac{15}{31}s^3$
$\quad \times\times$



(16) given RC admittance $Y(s) = \dfrac{11+2s+s^2}{12+5s+s^2}$

is not a proper admittance f^n

($\because$ The residues of the poles must be real and +ve)

hence not realizable.